MODELLING OF THE PROPAGATION THROUGH A CIRCULAR DIELECTRIC WAVEGUIDE

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ABSTRACT

In this thesis, effect of electric and magnetic field that propagates throughout propagation direction separately or together are researched. When fields are propagated separately, formation of symmetrical transverse magnetic (TM) or transverse electric (TM) modes are researched on circular dielectric waveguide and also this is done this is done when the both of electric and magnetic fields are propagates throughout propagation direction, creation of EH and HE modes are researched. In addition, weakly circular dielectric waveguide that core and cladding relative dielectric constant value is near each other and non-weakly circular dielectric waveguide that core and cladding relative dielectric constant is different than each other are investigated. Hence, critical angle value and angle of modes with changing frequency, normalized propagation constant value and electric and magnetic field distribution are observed on the waveguide and differences between the weakly guiding and non-weakly guiding fiber are tried to find.

INTRODUCTION

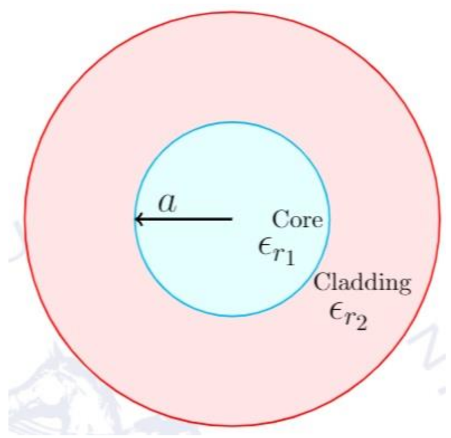


Figure 1: Structure of Cable

Inner and outer circle represent core and cladding of optical fiber cable. In addition, ε_{r1} and ε_{r2} represent the relative dielectric constant of core and cladding. Inner radius is 10µm.
To find existence of all kind of modes, Helmholtz's wave equation

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0 \qquad \nabla^2 \vec{H} + k^2 \vec{H} = 0$$

and Maxwell's equation are used.

$$\nabla \times \vec{E} = -jw\mu\vec{H} \qquad \nabla \times \vec{H} = jw\epsilon\vec{E}$$

- **TRANSVERSE MAGNETIC FIELD**

In this mode, fields in propagation direction $e_z(\rho,\varphi) \neq 0$ and $h_z(\rho,\varphi) = 0$. Via of Helmholtz wave equation;

$$E_z = \begin{cases} E_{core} J_0(k_{core}\rho) & \rho < a \\ E_{cladding} K_0(\alpha_2 \rho) & \rho > a \end{cases}$$

From Maxwell's equation;

$$\begin{vmatrix} J_0(ak_{core}) & -K_0(a\alpha_2) \\ \frac{\epsilon_{r1}}{k_{core}} J_0'(ak_{core}) & \frac{\epsilon_{r2}}{\alpha_2} K_0'(a\alpha_2) \end{vmatrix} \cdot \begin{vmatrix} E_{core} \\ E_{cladding} \end{vmatrix} = 0$$

- **TRANSVERSE ELECTRIC FIELD**

In this mode, fields in propagation direction $e_z(\rho,\varphi)=0$ and $h_z(\rho,\varphi) \neq 0$. . Via of Helmholtz wave equation;

$$H_z = \begin{cases} H_{core} J_0(k_{core}\rho) & \rho < a \\ H_{cladding} K_0(\alpha_2 \rho) & \rho > a \end{cases}$$

From Maxwell's equation;

$$\begin{vmatrix} \frac{\mu}{k_{core}} J_0'(ak_{core}) & \frac{\mu}{\alpha_2} K_0'(a\alpha_2) \\ J_0(ak_{core}) & -K_0(a\alpha_2) \end{vmatrix} \cdot \begin{vmatrix} H_{core} \\ H_{cladding} \end{vmatrix} = 0$$

- **HYBRID MODE**

In this mode, fields in propagation direction $e_z(\rho,\varphi) \neq 0$ and $h_z(\rho,\varphi) \neq 0$. Via of Helmholtz wave equation;

$$E_z = \begin{cases} E^e_{core} J_n(k_{core}\rho) e^{jn\phi} & \rho < a \\ E^e_{cladding} K_n(\alpha_2 \rho) e^{jn\phi} & \rho > a \end{cases}$$

$$H_z = \begin{cases} H^h_{core} J_n(k_{core}\rho) e^{jn\phi} & \rho < a \\ H^h_{cladding} K_n(\alpha_2 \rho) e^{jn\phi} & \rho > a \end{cases}$$

From Maxwell's equation;

$$\begin{vmatrix} \frac{n\beta}{ak^2_{core}} J_n(ak_{core}) & \frac{n\beta}{a\alpha_2^2} K_n(a\alpha_2) & j\frac{w\mu}{k_{core}} J_n'(ak_{core}) & j\frac{w\mu}{\alpha_2} K_n'(a\alpha_2) \\ J_n(ak_{core}) & -K_n(a\alpha_2) & 0 & 0 \\ -j\frac{w\epsilon_{r1}}{k_{core}} J_n'(ak_{core}) & -j\frac{w\epsilon_{r2}}{\alpha_2} K_n'(a\alpha_2) & \frac{n\beta}{ak^2_{core}} J_n(ak_{core}) & \frac{n\beta}{a\alpha_2^2} K_n(a\alpha_2) \\ 0 & 0 & J_n(ak_{core}) & -K_n(a\alpha_2) \end{vmatrix} \cdot \begin{vmatrix} E^e_{core} \\ E^e_{cladding} \\ H^h_{core} \\ H^h_{cladding} \end{vmatrix} = 0$$

WEAKLY GUIDING FIBER

- **TRANSVERE MAGNETIC FIELD**

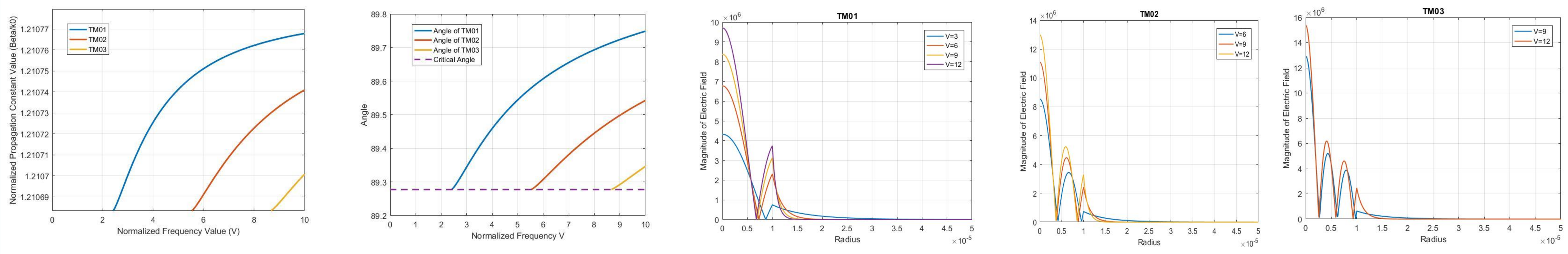


Figure 2. Propagation Constant , Angle, and Electric Field Distribution of TM

- **TRANSVERSE ELECTRIC FIELD**

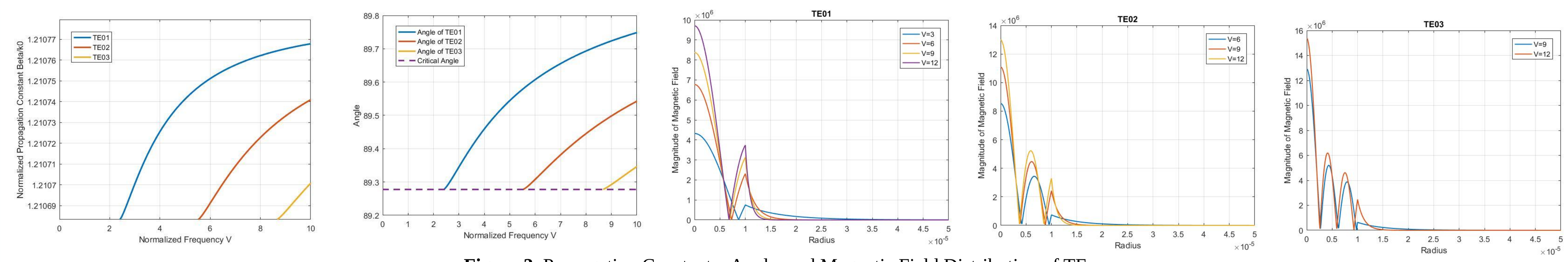


Figure 3. Propagation Constant , Angle, and Magnetic Field Distribution of TE

- **HYBRID MODES**

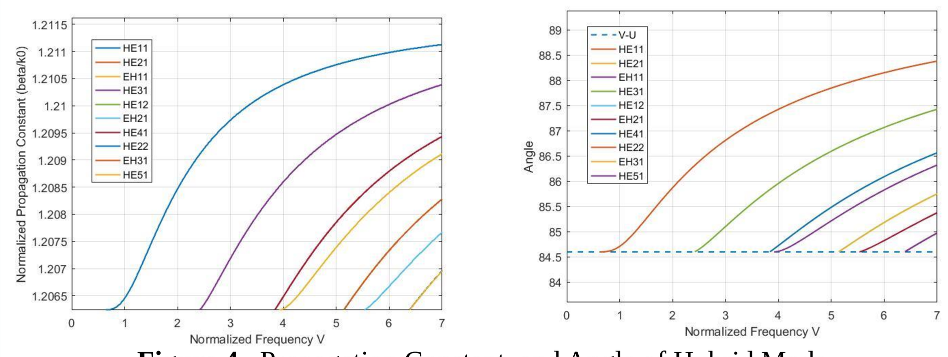


Figure 4. Propagation Constant and Angle of Hybrid Mode

In weakly guiding optical fiber cable, differences between the reactive indexes of core and cladding region should be less than 10^{-4}. Because of this reason, ε_{r1} and ε_{r2} should be 1.465987 and 1.465754 respectively.

NON-WEAKLY GUIDING FIBER

- **TRANSVERSE MAGNETIC FIELD**

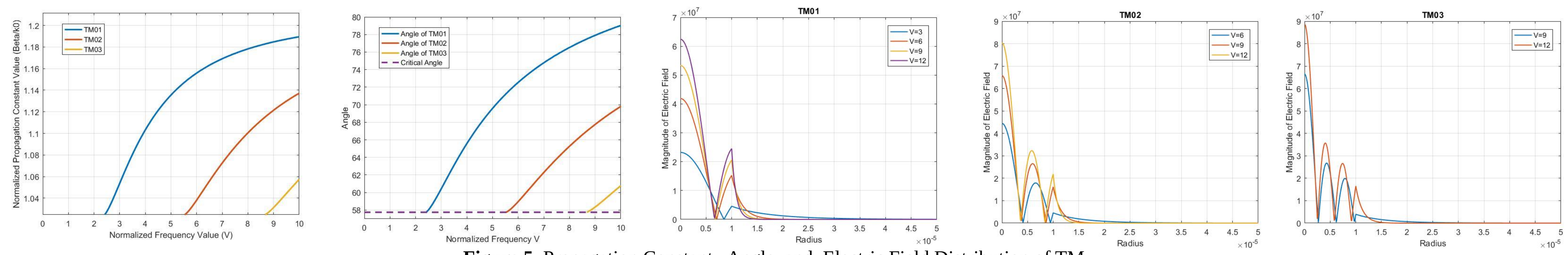


Figure 5. Propagation Constant , Angle, and Electric Field Distribution of TM

- **TRANSVERSE ELECTRIC FIELD**

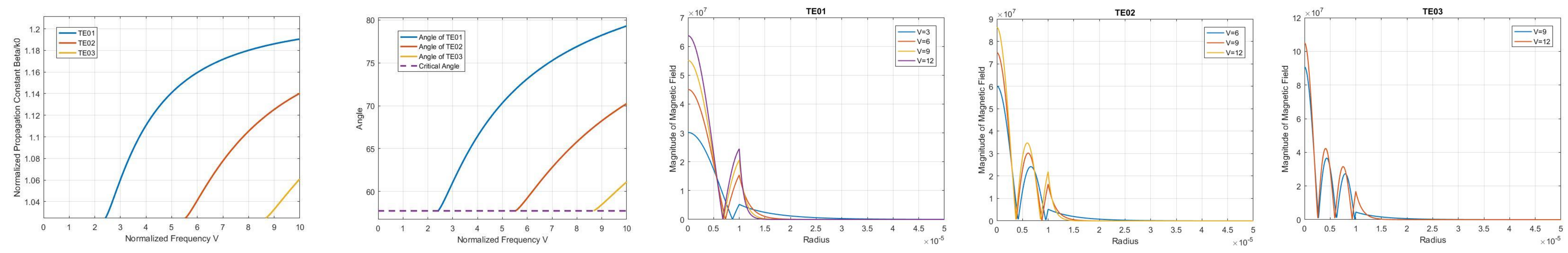
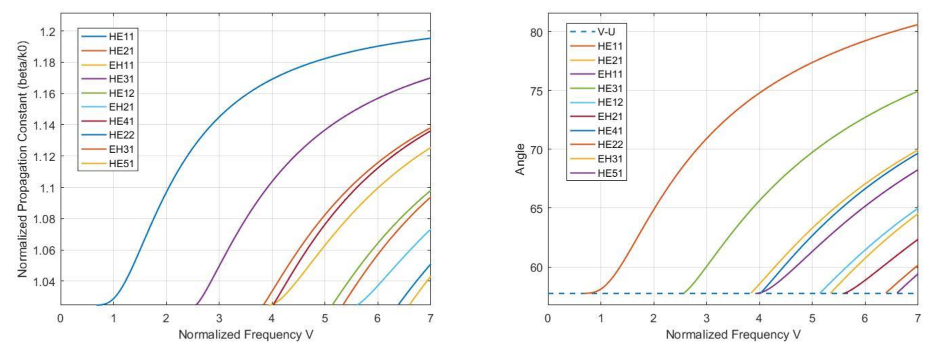


Figure 6. Propagation Constant , Angle and Electric Field Distribution of TE

- **HYBRID MODE**



In non-weakly guiding optical fiber cable, differences between the reactive indexes of core and cladding region should be bigger than 10^{-4}. Because of this reason, ε_{r1} and ε_{r2} should be 1.468 and 1.05 respectively.

Figure 7. Propagation Constant and Angle of Hybrid Mode

WEAKLY GUIDING FIBER VS. NON-WEAKLY GUIDING FIBER

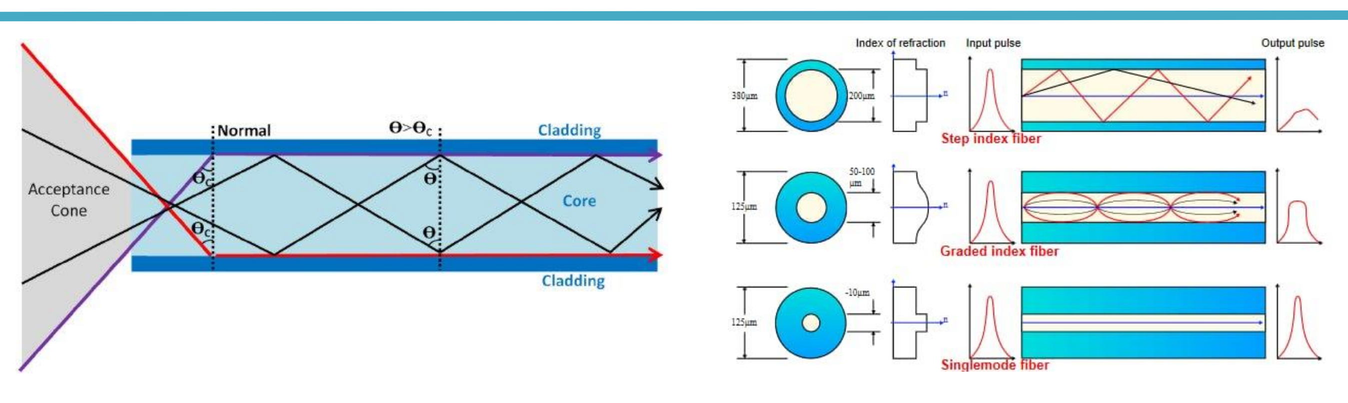


Figure 8. Acceptance cone and dispersion of circular dielectric waveguide

- Critical angle on the non-weakly guiding fiber is about 58° and critical angle on the weakly guiding fiber is about 89°.
- Because of this differences between the critical angle of optical fiber types we can say that acceptance cone of non-weakly guiding fiber is bigger than acceptance cone of weakly guiding fiber.
- In addition, modal dispersion of non-weakly guiding fiber is much more than modal dispersion of weakly guiding fiber

- In non-weakly guiding fiber,magnitude of electric or magnetic field on boundary is bigger than weakly guiding fiber's.
- Decrement of electric or magnetic field of non-weakly guiding fiber in the cladding region again more than decrement of weakly guiding fiber
- Attenuation of optical fiber cable is less than attenuation of coaxial cable
- Normalized frequaency value depends on the relative dielectric constant of core and cladding and Radius of the core. Thus, with the increment of Radius of core, normalized frequency is increasing and we can observe the too much mode on the optical fiber cable.
- There is a waveguide dispersion on both of weakly guiding fiber and non-weakly guiding fiber.

CONLUSION

The purpose of this research mathematical analysis and numerical simulations are performed for circular dielectric waveguide. When only electric or magnetic field propagates through z direction, TM and TE modes are observed. If both of electric and magnetic field are propagates throughout z direction, hybrid modes EH and HE are observed. In addition, TM , TE and hybrid modes are observed on the changing frequency. However, number of modes in waveguide is changed with frequency. This means that there can be more than one mode at the same time in the circular dielectric waveguide. Additionally, when the frequency of source is exceed the cutoff frequency of modes, symmetrical mode TE and TM and hybrid mode HE and EH start to propagate.
Secondly, we observe that propagation constant of all modes change between the ε_{r1} and ε_{r2}. In the weakly guiding fiber, the lowest and highest propagation constant of modes and angle are close to each other. Thus, modal dispersion on the weakly guiding fiber is low. In the non-weakly guiding fiber , lowest and highest normalized propagation constant of modes and angle are different than each other. Modal dispersion on the non-weakly is high when we compare it weakly guiding fiber.
Additionally, on the cladding region, reduction speed of magnitude of field in the non-weakly guiding fiber is much more than reduction speed of magnitude of field in the weakly guiding fiber. In addition, inside the core region, magnitude of electric or magnetic field intensity increase with the increment of the frequency and also magnitude of electric or magnetic field intensity of non-weakly guiding fiber is much more than magnitude of electric or magnetic field intensity of weakly guiding fiber.